

Ein- und Mehrteil-Verteilungsfunktionen vorgeschlagen, und eingeführt wird ein Verhältnis zwischen statistischer und dynamischer Auffassungen in der Theorie der Übertragung.

Eines der letzten Ergebnisse von E. W. Tolubinsky besteht in der Entwicklung des  $[S]$ -Operators, vermittels dessen man der willkürlichen  $\psi(\vec{r}, \vec{v})$ -Funktion aus einem gewissen Funktionalraum eine solche  $\rho(t) = [S]\psi$ -Dichte gegenüberstellen kann, dass  $\rho(t)d\sigma$  die Wahrscheinlichkeit des aufgefundenen Systems im Zustand vorstellen wird, der durch die Einteil-Verteilungsfunktion von  $\psi(\vec{r}, \vec{v})$  gekennzeichnet wird, wobei  $d\sigma$  ein Volumelement im Funktionalraum ist. Die Art des  $[S]$ -Operators ist von wesentlichem Interesse für die Forschung der zeitlichen Systemevolution sowie für das Studium der Eigenschaften stationärer Gleichgewichtsverteilungen.

Die Werke von E. W. Tolubinsky zählen zu den hervorragenden Errungenschaften auf einem der rein begrifflichen, schwierigen und wichtigen Gebiete der Physik. Deshalb ist kein Zweifel, dass die in der Monographie dargelegten Ergebnisse weite Verwendung finden werden.

Die lineare Theorie der Übertragung und insbesondere das Umrechnungsverfahren werden zur Untersuchung der Übertragungsvorgänge in verdünnten Gasen verwendet.

Die tiefen Ideen, die Grundlagen der in-linearen Theorie der Übertragung schaffen, besitzen grosse Möglichkeiten zur Forschung von kondensierten Systemen (dichte Gase und Flüssigkeiten).

Bemerkenswert ist der Reichtum der Idee und der Ergebnisse der Monographie von E. W. Tolubinsky—geschweige solcher fundamentaler Ergebnisse, wie das Umrechnungsverfahren und der Berechnungsalgorithmus der dort entstandenen Integralen über die Flugbahnen; über die Grundlagen der in-linearen Theorie der Übertragung, die vom hervorragenden Talent ihres Verfassers zeugen. Viele wichtige Ergebnisse von E. W. Tolubinsky würden als ob nebenbei bestimmt. Es genügt zu erwähnen den unabhängigen Beweis der Ergodenhypothese, begründet auf der Beibringung der metrischen Transitivität, sowie die Formulierung der Anfangsbedingung in der Randaufgabe für die hyperbolische Übertragungsgleichung, die zur Erfüllung des Gesetzes der Erhaltung von übertragenden Substanzen nötig ist.

Wegen des begrenzten Umfanges dieser Rezension konnten mehrere Ergebnisse überhaupt nicht erwähnt werden.

Die ausgezeichnete Untersuchungen von E. W. Tolubinsky abschätzend, kann behauptet werden, dass das von ihm formulierte Programm zum Aufbau der Theorie der Übertragungsphänomene, die erhaltenen Ergebnisse und die von ihm prinzipiell neu entwickelten Verfahren ohne Zweifel als Grundlage für die Entwicklung einer neuen wissenschaftlichen Richtung in der statistischen Physik der Ungleichgewichtsvorgänge dienen werden.

T. L. PERELMAN

**One-dimensional Two-phase Flow**, GRAHAM B. WALLIS.  
McGraw-Hill, New York (1969).

SOMETIMES restrictions can be liberating. Resolve to put all distant projects out of mind, accomplish to-day's work to-day, and leave behind an empty desk; and on the morrow

those deferred projects will be waiting for you, no longer as remote, unclear aspirations, but as attainable goals of the immediate future. Professor Wallis has performed just such a hygienic labour. Restricting himself religiously to *one-dimensional* flow of two-phase mixtures, and resolving to write all that is useful about them in the clearest possible way, he has made one question obvious: Now that's finished, what about *two-dimensional* flows? For he has done his job well: careful with his definitions, systematic and comprehensive in his organisation of material, copious and vivid in his choice of illustration, and forthright in dispensing advice, he has produced a book of substantial finality.

What then does prevent the launching of a large-scale attack on two-phase flows in two dimensions? Certainly not lack of practical importance; for the immense capital investment in equipment embodying two-phase pipe flow, and the difficulty of predicting its performance without massive experimentation, would justify the development of a theoretical design procedure, even at the cost of some hundreds of thousands of dollars. Is it then the mathematical difficulty of solving the differential equations that would be needed to describe the behaviour of a realistic mathematical model? Again, no; for there now exist standard computer programs for solving sets of simultaneous non-linear parabolic and hyperbolic differential equations; and these can certainly be adapted to the solution of the particular equations which are appropriate to two-phase mixtures.

A more serious obstacle to progress is the lack of quantitative knowledge of some important physical processes, for example the momentum interchange between the two coexistent phases. Yet many of these processes take place in one-dimensional flows also, and Professor Wallis has now catalogued them for other processes, it is possible at least to make plausible first guesses. So it seems that one could now write down the set of differential and algebraic equations which govern the velocities, concentrations and other properties of the mixture in two dimensions; and these equations could be solved with ease. Surely systematic comparison of the resulting solutions with available experimental data would enable the first guesses to be refined, and the necessary new hypotheses to be invented?

If economic, mathematical and physical obstacles are so easily removed, only psychological ones can still impede progress: the possibility of advance has been too murky perceived, and its prospects of success too pessimistically rated. Professor Wallis's book has cleared away much of the obscurity; it will be a valuable guide into new areas of research and application.

D. B. SPALDING

**Diffusion and Heat Transfer in Chemical Kinetics**, D. A. FRANK-KAMENETSKII.

EVERYONE who has studied heat or mass transfer for systems that involve chemical reactions knows Frank-Kamenetskii's classical monograph "Diffusion and Heat Exchange in Chemical Kinetics". Knowing also that it has become difficult to obtain copies of that work, they will welcome this translation of the newly prepared (1967) second edition. The viewpoint of the second edition is the same as that of the

first, and the same ten chapters appear. But the content has been vastly expanded and updated. The new volume contains more than one and a half times as many pages and, because of a larger page size, nearly twice the material. The extensions involve pedagogical improvements, through explanations that are more thorough, and additions of a great amount of material of basic importance, that was unavailable in 1947 when the first edition was prepared. Moreover, the new translation is excellent; it is much better than the first, which itself was reasonably good. In the new volume, particularly in early chapters, one must search carefully to find unusual English phrases (typical examples being "completely stabilized flow" instead of "fully developed flow" and "inflammation" instead of "explosion"). The editor and publisher are to be congratulated for bringing forth so readable a translation so soon after the appearance of the Russian publication.

The content reflects the interests of the author. A number of the concepts and some associated terminology are unique. The theme of transfer processes involving chemical kinetics is developed with heterogeneous reactions emphasized more than homogeneous reactions, low-speed flow much more than high-speed flow, approximate methods more than exact calculations, limiting regimes such as diffusion-controlled and kinetic-controlled behaviour more than intermediate cases, qualitative concepts such as ignition, extinction and oscillatory reactors more than detailed studies of concrete systems, theory more than experiment. Yet, there is good coverage even of most topics that are not emphasized. Because of the peculiar arrangement of material, it is difficult to find specific items in the volume; for example, discussions of carbon combustion are scattered in at least four widely separated sections. However, the excellent index helps to mitigate this difficulty. Upon starting a chapter, the reader gains little knowledge of his destination. Only after wading through detailed derivations does he discover where he has been led. The road is long but traversed gradually enough for the most backward reader to follow, if he has sufficient patience. In retrospect one finds a marked coherence to the volume, not only within chapters but often between chapters. Were Dostoevsky a technologist, I think that he would write a book like this. Even remarks and opinions that are earthy occasionally occur. The effort required to study the book will be well spent because of the improved insight that will ensue. While it is not an elementary textbook, the volume is quite instructive to the mature reader.

It may be of interest to indicate, without detailed critical comment, some highlights of the second edition. A thorough exposition and defense are presented for the author's "method of the equiaccessible surface", wherein transport and reaction are separated for the purpose of simplifying the analysis of heterogeneous kinetics. The specific techniques described can prove quite helpful to those concerned with calculating transport processes for complicated systems with heterogeneous reactions. A thorough presentation is given of the method of Laplace transforms for solving diffusion problems with various boundary conditions. Concepts concerning Stephan flow are updated on the basis of results from the near-equilibrium molecular theory of gases. All of the essential results of the exact kinetic

theory for diffusion and thermal diffusion of multicomponent ideal gas mixtures are derived from the exceedingly useful elementary viewpoint of momentum conservation applied separately to each of the mutually interacting species. Analyses are given for concentration fields in both laminar and turbulent boundary layers with high Schmidt numbers. The complete basis of combustion theory is set forth succinctly, with minimum computation and maximum insight. Thorough expositions are given for thermal explosion theory and for flame propagation theory with both one-step and complex kinetics. Steady-state and transient theories are developed for ignition and extinction of heterogeneous reactions. On the basis of phase-plane analyses, critical conditions are derived for the occurrence of both chemical kinetic oscillations and relaxation oscillations in a variety of closed and steady-flow reacting systems. It can be seen from this list that the book will interest those who specialize in heat and mass transfer, in combustion and in chemical engineering.

FORMAN A. WILLIAMS

**Variational Principles in Heat Transfer**, M. A. BIOT, 185 pp.  
Clarendon Press: Oxford University Press, 1970.

WHEN teaching heat and mass transfer to graduate students, instructors have often wished to have some convenient monograph on variational methods to which students can be directed. Variational methods have never occupied in heat transfer the significant role they have in classical mechanics, and it is probable that a student's introduction to variational methods comes from study in classical mechanics. This is desirable for readers who pick up this book, because Dr. Biot plunges immediately into the fundamental variational principle in heat conduction with a style and notation familiar in Lagrangian mechanics. The text is based upon about twenty papers by Dr. Biot, cited about 50 times in the 180 pages of text. Persons familiar with the original papers will find a useful reordering of the material; following the fundamental variational principle, successive chapters treat the theory of linear systems (including orthogonality and normal coordinates), operational formulation, associated fields, non-linear systems, convective heat transfer, boundary-layer heat transfer and complementary principles. An appendix sketches treatment of related subjects, including mass transport and irreversible thermodynamics. Despite the all-embracing "heat transfer" in the title, radiation is considered only peripherally.

An attraction of variational principles in physical sciences is that they provide convenient vehicles for approximate analysis, where approximations for, say, temperature distributions are written down so as to represent anticipated characteristic features of a solution in terms of a small number of parameters which take the role of generalized coordinates. Approximate solutions can be achieved quite quickly. A disadvantage is that the accuracy of such an approximate solution does not lend itself to investigation in a formal way.

The book is a personal monograph in the Oxford Mathematical Monograph series. It is concerned principally with